Learnings from using Geogebra to support mathematics teaching in government high schools

Radha Narve, Government high School, Beguru &
Sriranjani Ranganathan, IT for Change

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1 Context

With the near universalization of elementary schooling, the number of first generation school-goers entering high school is increasing. Children come to the 8^{th} standard in Government high schools with different learning experiences, from different higher primary school environments, different socio economic backgrounds , different mediums of instruction, etc. This creates an enormous heterogeneity of students at the entry level in a high school. This makes it a challenge for the teacher to understand the students first, their learning levels and learning abilities.

On the other hand, the education system has set high academic expectations by setting the same syllabus and text books, which assume a certain degree of academic proficiency. The syllabus and the content of the textbook are not often very relevant to the students from different social and economic backgrounds. The learning gaps, in literacy and numeracy, that students have when they enter high school has been documented[1]. The gap between the students' learning levels and skills and the academic expectations presents a real practical challenge, which a high school teacher is expected to address within three years of high school period.

1 ASER Education Report

Mathematics teaching has an additional challenge in terms of the language. The child's mother tongue will be different from the medium instruction of the school. Even a child with good ability to grasp concepts often fails to express or communicate properly because of difficulties of language. The ability to think and express in formal ways (which is what spoken language is) may be developed to different degrees in children. The formalism of mathematics language and expressions may not correspond with the spoken language. Even if they are able to express the spoken language, translating to math-speak poses a difficulty for many students. Even if the child is literate, meaning able to read, there may be difficulty with mathematical thinking and expression.

These constraints are exacerbated in the case of urban government high schools where students come from very impoverished backgrounds. The stratification of students who enter government schools is very high in cities as compared to rural areas. This results in a classroom environment that self selects downwards. The poverty of their home environment as well as the attractions of the city may propel the student towards work than school. With a syllabus that seems to be not relevant and faced with inadequate learning skills many students simply resist the school and the academic program. They are often disengaged from the classroom activities.

It is in this overall context that this paper examines the effectiveness of Geogebra use in mathematics classrooms.

2 Approach of the paper

This paper is a case study of the observations in four government high schools in Bengaluru South district. The authors have worked with students and teachers in these four government high schools. Teacher experiences and student activities from using Geogebra in these classrooms is the primary source of the discussion. To understand the experiences of mathematics teachers in using Geogebra, responses were also collected from a group of experienced teachers across the state. These teachers were identified based on their participation and sharing of various mathematics resources including Geogebra, in the online forum of the state.

The paper attempts to analyze the experiences of Geogebra use in terms of the achievement of different skills of mathematics learning and compares the effectiveness of Geogebra with other traditional methods of mathematics teaching learning. This is done through a description of lessons on specific concepts; the focus is on the learning processes possible than on the development of the mathematical idea. The ideas presented are illustrative and not exhaustive of all the classroom interventions. This paper also attempts to identify the teacher development trajectory possible through the use of a "constructionist[2]" tool like Geogebra and points to some challenges to the adoption of Geogebra as a method of mathematics teaching learning.

3 A brief introduction to Geogebra

Geogebra is a graphics software application, this enables a teacher to create lessons and resources which can be used for teaching-learning. It is a free educational application and and has its own respository of files, http://Geogebratube.org, available as Open Educational Resources. It can be downloaded freely (from http://sourceforge.net/projects/Geogebra) and it is available on GNU/Linux and Windows platrforms.

Geogebra allows you to make sketches on a graphic view and the algebra pane describes the co-ordinates of the construction. Functions can also be defined using the input bar. It is possible to animate constructions in Geogebra by defining the variable as a slider. Geogebra also allows instruction to be typed in Indic languages. The graphics view of Geogebra can be viewed in 2 dimensions or 3 dimensions. The tool also has a spreadsheet option to work with list values for plotting a function.

Geogebra construction follows the logic of mathematical constructions and the tool bar, will work in the same way, a paper and pencil construction would work. The construction protocol can be viewed by anyone using a Geogebra file. This is useful for the teacher and student to make comparisons between the two processes.

4 Development of mathematical competencies

Different curricula[3] have identified key competencies that should result from a mathematics teaching learning program. These include conceptual understanding, procedural fluency, problem solving, reasoning and logic and communicating and engaging with mathematics. We examine in this paper how use of Geogebra can support the attainment of these competencies.

4.1 Conceptual understanding

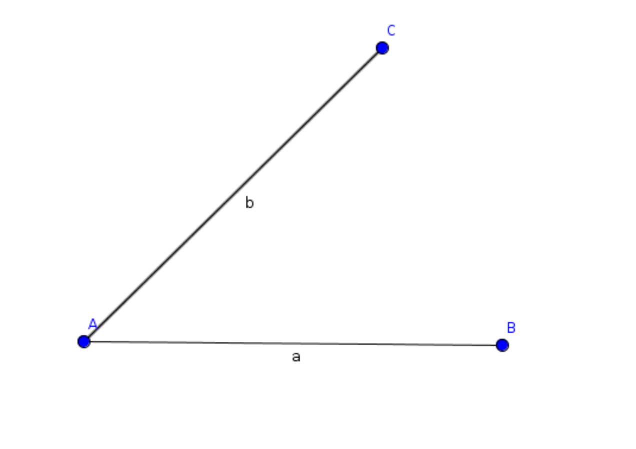
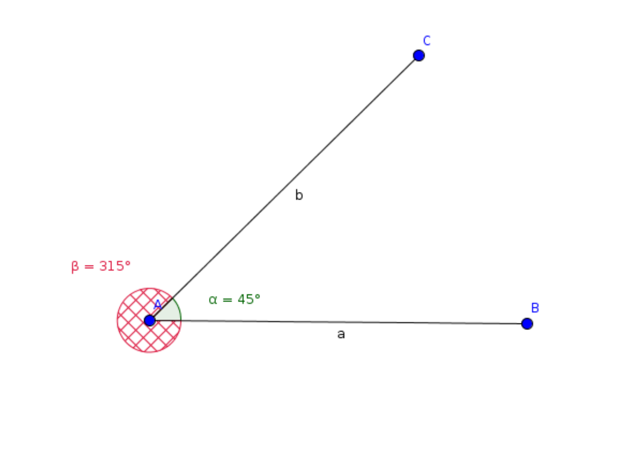
Students entering class 8 often have many conceptual gaps in their mathematics understanding. A pre-test at Class 8 level revealed that many students had difficulty identifying and naming basic concepts in geometry. Students have several misconceptions around such basic concepts as measuring angles, naming angles, properties of angles and lines. Students tend to memorise many

2 Seymour Paper on constructionism as a method of learning using digital artefacts
3 Adding it up: Helping children learn mathematics (National AcademicPress, 2001) and the Position paper on Teaching of Mathematics (NCF, 2005)

properties of plane figures and often are not able to visualize the geometry. For example, students will be able to determine if two angles are complementary, by adding the numbers, but would often not be able to demonstrate that two complementary angles placed next to each other will make a right angle. To help students understand the geometric concepts, a visualisation tool can be of great help. While hands-on activities have often been used for illustrating several geometric concepts, abstraction from the activity to formal mathematics expression has been a challenge in mathematics classes. Geogebra, with the features of accurate visualization combined with animation can be a useful way for bridging this gap.

Methods of angle measurement

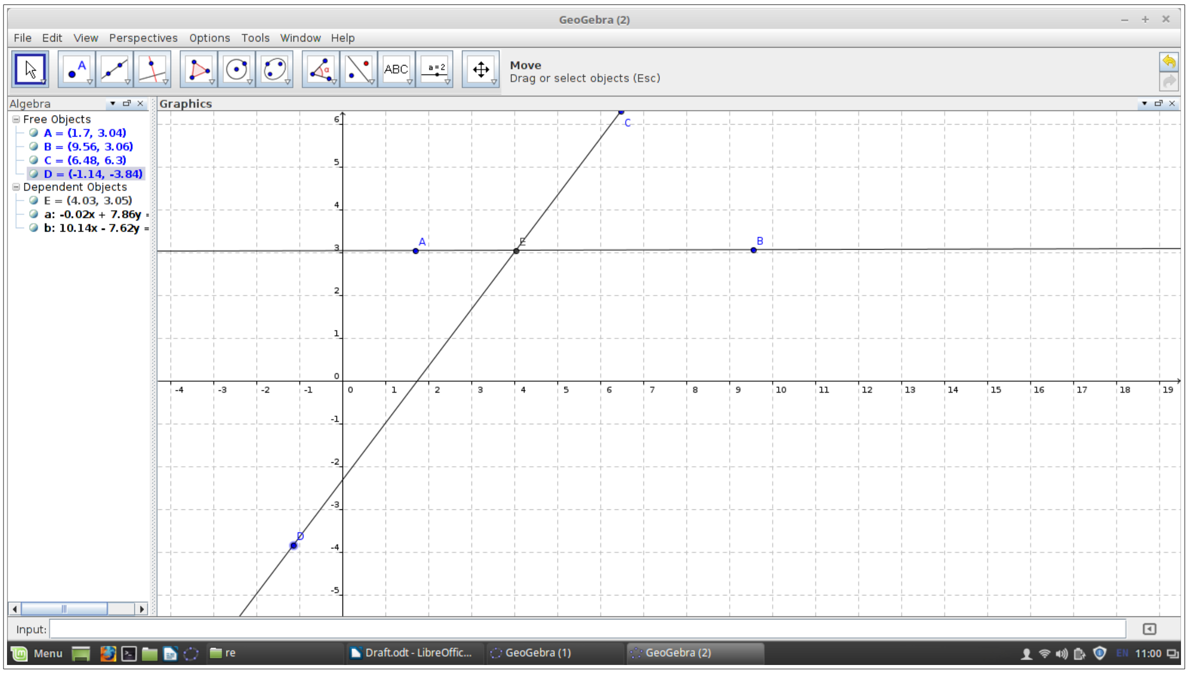
Students identify angles in different ways – when they see two line segments or see three points. They also commonly express that a longer arm of the angle means a larger angle. Often they do not see an angle as a rotation of an angle; in other words, formation of an angle is not understood.

	
Students identify that an angle has been formed. They also believe that if the segments AB and AC are made longer, the angle will be larger.	In the second question, students are asked to measure the angle after showing them the angle measuring tool. They say B,A,C or C,A,B, and are surprised to see two angle measures.

The concept of how an angle is formed between two segments can be discussed and reinforced by animating this construction, using a slider. This becomes an useful point to introduce to students the direction of rotation as a determinant of angle measure and that, by convention, we measure angles counter clockwise. Geogebra gives an option to measure in both ways, allowing the teacher to contrast and show. This notion of angle measurement is often not given adequate importance in the younger classes but becomes a source of confusion when co-ordinate geometry or trignometry is introduced. This lesson can be further extended to talk about the angles formed by intersection of two lines. After having understood the concept of measurement of angles, it becomes easier for students to identify angles formed by intersecting lines, angles formed by rotating rays and so on.

Vertically opposite angles are equal

Even as they get confused about how to measure and label an angle, students will be able to identify vertically opposite angles quite accurately, supported by memorization. They will, however, not be able to explain why vertically opposite angles are equal.



If the students are asked how many angles are formed, they will readily say two angles are formed. More perceptive students will say 4. They will also identify one pair of vertically opposite angles and, on some questioning, they may be able to identify two pairs. As these angles are formed by the intersection of two lines (or line segments), we can choose two intersecting lines, and then mark the point of their intersection . Here, the teacher could face a challenge when one of the line is made to rotate (as an animation) to show the change in angles. As the intersection point changes, it could disturb the visualization of the students. The teacher could either fix the line (and the intersection point) or use the opportunity to discuss that vertically opposite angles are formed whenever two lines intersect.

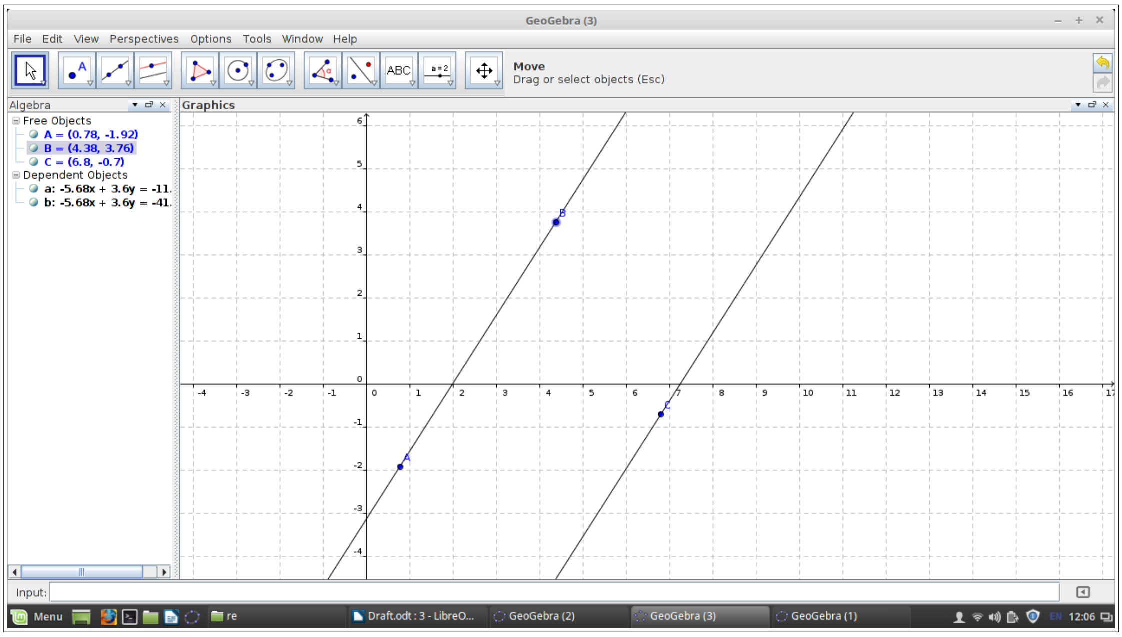
A further exploration can be to establish why vertically opposite angles are equal. After labeling all the angles (the measurement of angle will come in useful here), students can measure how many straight angles are formed. Again, with some questioning, students will say 4. It will be a useful lesson to add all the individual angles formed to discuss angle around a point and also discuss why all the straight angles cannot be added. Through these discussions, students are able to conceptualize vertically opposite angles better. In any lesson, there are related lessons. This lesson can also be used to demonstrate that minimum two points are needed to create a line.

4.2 Engaging the learner

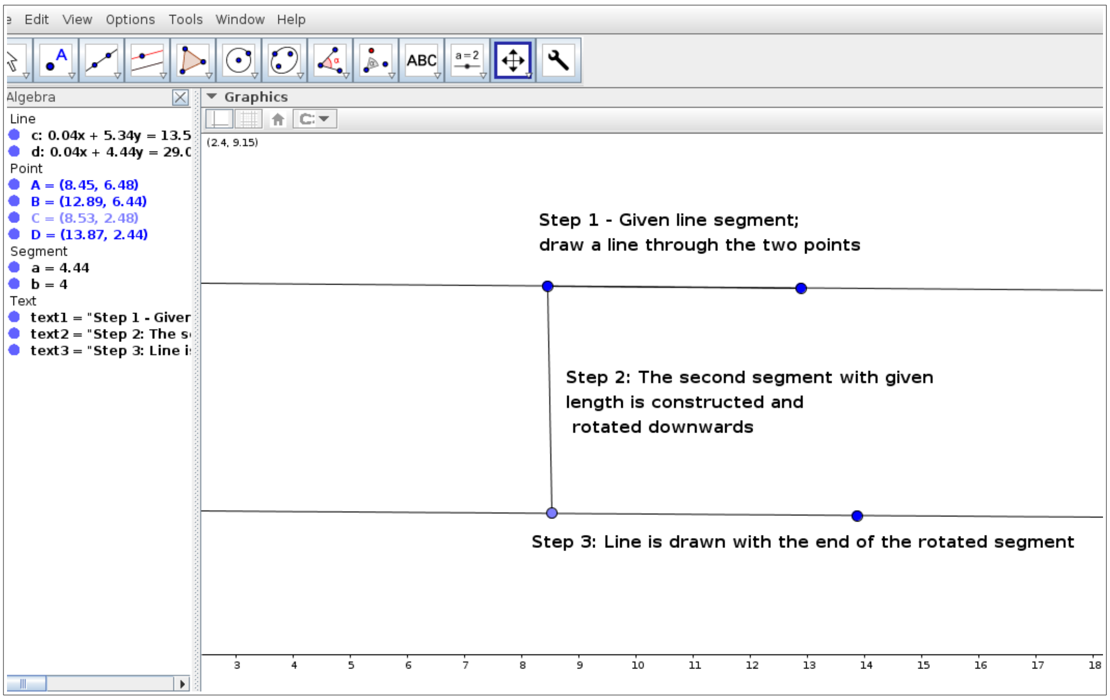
Students in mathematics classes are often disengaged. They do not seem to see the relevance of the discussions to them. Geogebra can be used to make a demand on the students to think.

Teaching parallel lines

When asked what are parallel lines, students would say window panes, railway lines etc but would often not be able to say why they are parallel. Creating parallel lines in Geogebra and showing that the distance between them does not change, how much ever we move along the lines, is a useful visualization to have. Rotating the lines and still showing that they are parallel is also useful.



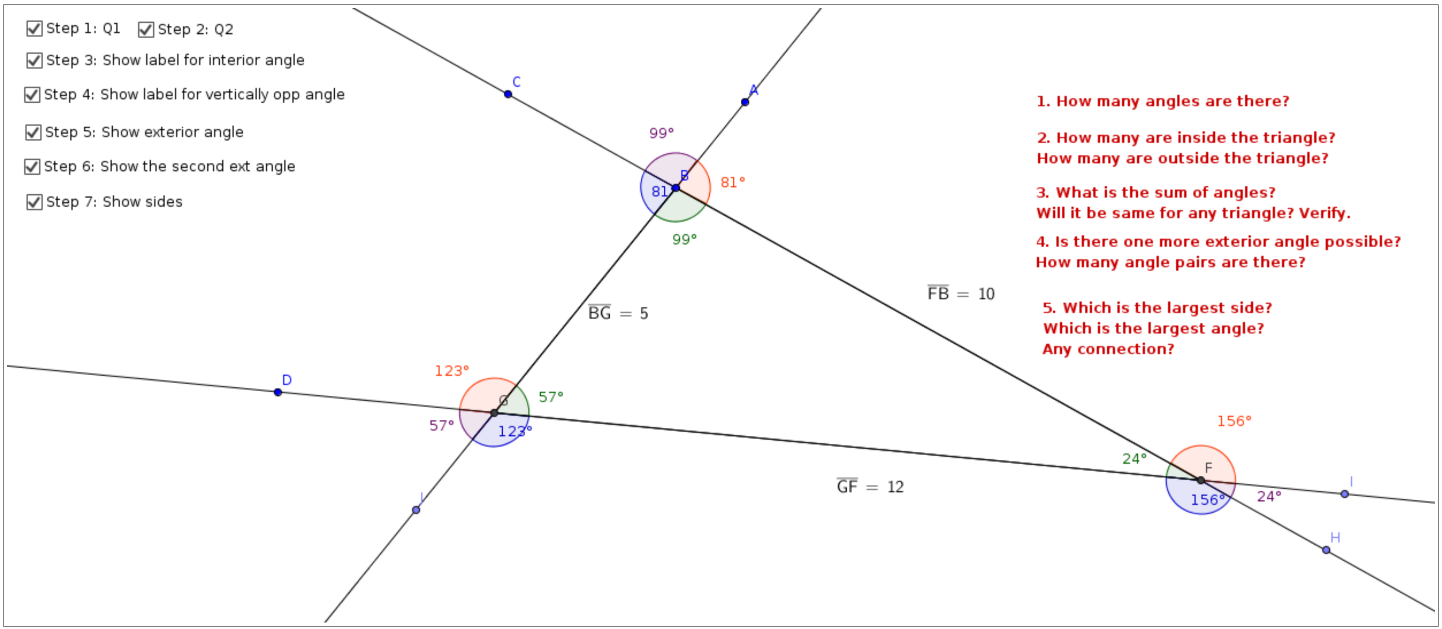
The following class was conducted on parallel lines in one of the classes. After having introduced the idea of parallel lines, students were asked to draw a parallel line to a given line segment at a fixed distance from the line segment. Many students suggested using Segment with given length option, rotating it and drawing a line through the rotated point to draw the parallel lines.



As you zoom out , it becomes possible for the students to see that this method may not give parallel lines. Evident from this discussion was the fact that students did not understand the notion of perpendicular distance. After further discussion, students were able to say the segment should be rotated down “straight”, meaning perpendicular. And through further discussions, the class constructed a line parallel to the given segment. The teacher demonstrated drawing a perpendicular from each end of the segment and using the circle feature to mark off the required distance on the perpendicular line to draw a parallel line, below the given segment. She was promptly asked, if the parallel line could be drawn only below the line segment or above as well. The students were also asked to examine if they could use the “segment rotation method” to construct a parallel line. Though the students could not figure out the answer, they came back and asked the teacher to explain the construction. The extent of participation, engagement and desire to complete is worth noting, What has also been observed in this and other such classes, when Geogebra is used to co-construct concepts, a high level of discussion is even seen among students. Once this method of constructing parallel lines has been established, properties of angles by a transversal with parallel lines can be discussed.

Teaching triangles

Another example of engaging students in learning was observed when teaching triangles. Traditionally a triangle is introduced by drawing three line segments. However, rich conversations were recorded when the triangle was introduced as the region formed by the intersection of three lines. It was very intersecting to introduce triangle as a planar region created by the intersection of three lines instead of three line segments joining at their end points as we do in normally while introducing to polygons. It helps to visualize the area bounded by the three intersecting lines which changes with the orientation of any line.



Some of the questions (and answers) are documented below:

How many angles are there? 3 and 6 were common answers. Very few students said 9 and 1 student in a class said 12. In another class, students named the angles only where there were alphabets resulting in very different responses. Some students counted the entire triangle as one angle. Students engage with the diagram and express their thinking, which helps a teacher to identify the misconceptions that they may have.

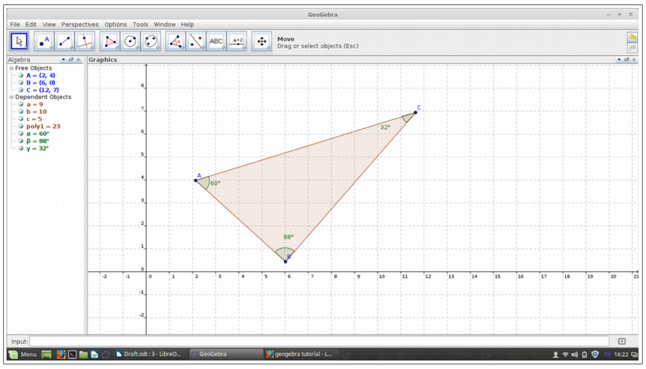
As an example, most of the children are unable to visualize the total number of angles formed in this diagram. Normally they will not be able to visualize exterior angles, as they often see only interior angles.

How many vertically opposite angles are there? Students usually say 3, suggesting again that their visualizations of angles is not very strong.

How many are inside the triangle? 3 was the answer.

How many are outside? Many students said 6. They were simply not able to visualize some angles as angles at all; often they were able to visualize only those angles which had been introduced to them as “exterior angles”. The lesson was extended to exterior angles at each vertex, the relationship between the angles and sides and so on.

Once the general idea of a triangle has been established like this, the triangle can also be introduced as three non-coinciding line segments, joined at the ends.



The student was allowed to think about what would happen if the measure of one of the angle is considered equal to zero degrees. Can a triangle have one angle more than 180 degrees? Can a triangle have two right angles? Questions and probes like these enable the teacher to make a demand on the students to think about mathematics and communicate.

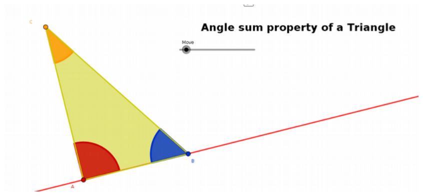
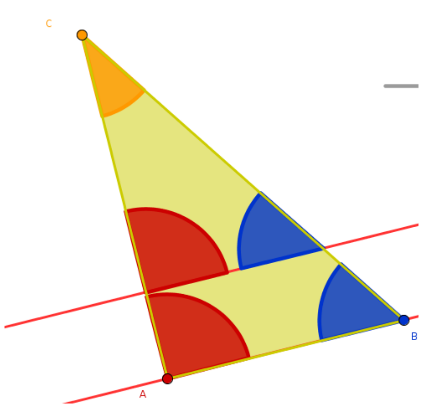
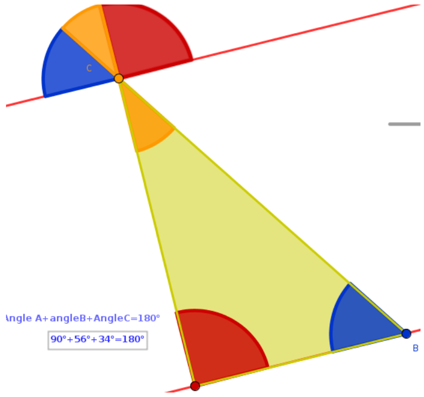
4.3 Procedural fluency

In all the examples discussed above, we saw that students had difficulties in visualizing, expressing their visualization in common language and further expression in mathematical terms. Using Geogebra to create a geometric construction in class it becomes possible to break the steps involved in a construction as well as allow students to explore their conceptions and mis-conceptions. This helps in their understanding the process of geometric construction better, unlike the traditional methods where students often memorize the construction steps. As we discussed earlier, the alignment of the Geogebra tool bar to the processes of physical construction makes it easier for students to articulate their understanding of how to construct. Geogebra has been able to support students as they work through the development of mathematical vocabulary.

4.4 Logical thinking and reasoning

Reasoning and using prior concepts to logically establish a mathematical truth is a key competency of any mathematics program. Quite often, students in government schools memorize theorems as they are unable to make the connections between steps in a logical proof. As can be seen from the previous discussion, gaps in conceptual understanding is one important reason why they are unable to work with logical proofs. Language competencies add to the difficulty.

Verifying a theorem and making Geogebra sketches for different steps of the logical proof can help students bridge the gap in thinking, helping students work the logical proofs. Dynamic feature of Geogebra allows a child to visualize the demonstration and understand the verified result.

		
A Geogebra sketch with the diagram of the triangle	An animation in progress of drawing a line parallel to one side	Using the properties of parallel lines established earlier, and known property of vertically opposite angles, the proof can be visualized and explained.

As an example, angle sum property can be verified by drawing a polygon and measuring its interior angles. But it is also important for a teacher to use Geogebra to invoke the logical reason for angle sum property of a triangle. Appropriate questions will allows a child to think about the need of a line parallel to one of the side of the triangle, sliding one of the side of a triangle along with the other two sides will be able to think logically how corresponding angles are same and with the prior knowledge about vertically opposite angles, angles on a straight line can establish the angle sum property of a triangle.

5 Impact on Teacher Professional Development

5.1 Constructionism as a mathematics pedagogy

Seymour Papert developed a learning theory called constructionism which extended the idea of constructivism. In his own words: "Constructionism—the N word as opposed to the V word—shares contructivism's view of learning as "building knowledge structures"through progressive internalization of actions... It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe (Papert, 1991, p.1). When students are working with Geogebra, they are creating digital aretfacts expressing their knowledge while at the same time working with the artefacts to build further knowledge[4]. Thus, with the use of Geogebra, the teacher can both look at the students' expressed knowledge as well as use for learning. This aspect makes Geogebra very suitable for developing formative assessments.

5.2 Designing assessments to support learning

A survey of 15 teachers using Geogebra indicates that more than 70% use it for assessments, either for creating digital outputs or in combination with printed worksheets. Geogebra can be used to set up learner investigations for problems or demonstrate steps of constructions, thus allowing the teacher to observe the students' progress and levels of conceptual understanding and problem solving. Use of Geogebra allows a teacher to set up assessments that can both evaluate technology learning as well as content learning. Students' quality of finished products and their processes of construction can show the extent of their technology learning.

5.3 Changing classroom processes

Teachers are traditionally used to instructing and telling concepts. It is also believed by many teachers that prior concepts have been understood. An inductive approach with students expressing their understanding is seen as a difficult approach to adopt, especially in the context of the learning gaps in high school as well as the syllabus load. In addition, given the lack of a medium and resource to facilitate an inquiry, teachers often used a demonstration approach to problem solving and reasoning. With the possibility of using Geogebra to facilitate an inquiry, there is scope for changing the classroom processes to include more inquiry based learning processes.

The usage of Geogebra creates a situation in which a student sees, visualizes, imagines, thinks and asks questions. This process of learning of the student can make a teacher to explore in depth, the deeper aspects of understanding a concept. Secondly, this process also enables a teacher to empathize with the child's thinking in this process. Understanding a student's thinking process is critical in mathematics teaching, since errors, wrong assumptions, conceptual gaps etc can be detected in this process, which would help a teacher to design teaching strategies.

5.4 Emergence of teachers' knowledge

When one of the authors, as a practising teacher, created her first applet, she assumed that the learning outcomes would change due to the mere creation of an applet with animation. However, as it has emerged in the paper, the creation of a Geogebra file in discussion with the class seemed to

4 Constructionism: Theory of Learning or Theory of Design, Chronis Kynigos, Educational Technology Lab, Department of Education, School of Philosophy, University of Athens

present greater pedagogic possibilities. While initially technological knowledge drives the use of Geogebra -in terms of creating more and more files to explain to students – over time, use of Geogebra allows the teacher to look at content differently as well as develop new pedagogies.

In addition to the 4 schools which were studied for the process, about 15 teachers across the state were surveyed on their practices, experiences and perspectives of Geogebra. Teachers with different teaching experience have found Geogebra useful. Many teachers in the survey conducted for this paper have reported creating Geogebra files in classes. They have reported being surprised and encouraged by student response to mathematics classes. The survey shows that 70% of the teachers creates their own Geogebra applets for the class room usage and they have their own Geogebra files library organized class wise and concept wise/chapter wise. More than 50 % of teachers survey reported using Geogebra used to teach all high school geometry concepts and some few of them used in revision, for arithmetic calculations and Geogebra as a worksheet too. More than 50% of the teachers have shared that the usage of Geogebra in the lesson sequence varies.

Many teachers have emphasized that Geogebra is not easy to use in classroom and requires a lot of subject understanding, technical understanding of using it along with a detailed plan and preparation before taking in to the classroom.

6 Conclusion

The interactive visual interface, precision construction and animation possibilities of Geogebra has helped students navigate language gaps that formal mathematics communication required, and provided advantages over physical models. Using the interface to replicate physical construction processes helped students to ‘do mathematics’, visualize mathematical possibilities and articulate their mathematical understanding and imagination. This supported movement towards logical proofs, including in strengthening learner reasoning to work independent of the specific diagrams drawn. The use of Geogebra by students to create artefacts supported assessment of their learning, by evidencing their thinking processes. Geogebra use has also supported teachers’ own journey of integration of technology in teaching learning.